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Passive Networks: Analysis of First and Second Order Circuits

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1 Contents

2 Introduction

Inductors and Capacitors produce a response to transient and frequency dependent signals that can be harnessed by engineers to produce certain circuit behaviours or condition circuit signals. This is due to their ability to store energy within a circuit. Circuits with a Capacitor or Inductor in series with a Resistor (RC or RL – First Order Circuits) can be used as electrical timers and filters due to their transient and frequency response respectively. Circuits with a Capacitor, Inductor and Resistor in series or parallel (RLC – Second Order Circuits) can be used as band frequency filters due to their response to a range of frequencies. These behaviours make First and Second Order circuits extremely useful signal conditioning applications such as oscilloscopes and audio amplifiers [1, pp. 48 - 50].

The Passive Networks experiment aims to analyse the transient and frequency response of First and Second Order circuits. This allows for greater understanding of the design and application of these circuits as well as further experience with Bode Plots, graphical data analysis and interpretation and comparison of data to theoretical models.

3 Theory

This section will discuss the theoretical equations and formulae required to complete the experiments detailed in this report. It explains the equations that were used to generate the theoretical data shown in section (INSERT SECTION NUMBER HERE) of this report. The frequency response of First and Second order circuits will be discussed, as well the transient response of First order circuits.

In AC circuits, Capacitors and Inductors have an Impedance that is frequency dependent, this is known as a reactive impedance. The impedance of an Inductor and Capacitor can be calculated using equations **3.1** and **3.2** respectively [1, p. 45].

$$Z_L = j\omega L \qquad 3.1$$

$$Z_C = \frac{1}{j\omega C} \qquad 3.2$$

These equations for impedance are the basis for all further theory shown in this section and can be used to describe the impedance of any circuit.

3.1 Gain

Gain is a measure of the ratio between the output signal and input signal of a given circuit. In this application it shows the amount of attenuation produced by the First or Second order circuit at a given frequency. Equation **3.3** was used to calculate the gain for all circuits in this report.

$$Gain\ (dB) = 20\log_{10}\left(\frac{V_{out}}{V_{in}}\right) \qquad 3.3$$

Plotting Gain against Frequency can show us details about what frequencies a filter will block, among other key information.

3.2 First Order Circuits – Frequency Response

These experiments show the frequency response of a Capacitor or Inductor in series with a Resistor. To theoretically characterise the response of these circuits to a frequency range we must first derive an equation for their impedance. The impedance of RL and RC circuits can be calculated using equations **3.4** and **3.5** respectively [1, pp. 48, 49].

$$Z_{RL} = \sqrt{R^2 + (\omega L)^2} \tag{3.4}$$

$$Z_{RC} = \sqrt{R^2 + \frac{1}{(\omega C)^2}} \tag{3.5}$$

These impedance equations can be used to calculate the voltage across components within the First order circuit. Consequently, they can be used to calculate the Gain for a specific First order circuit using equation **3.3**. This report calculates the Gain using the Resistor Voltage as V_{out} and the Supply Voltage as V_{in}. Equations **3.6** and **3.7** show how V_{out} can be calculated for RL and RC circuits respectively [1, pp. 48, 49, 51].

$$V_{out} = V_{in}\frac{R}{\sqrt{R^2 + (\omega L)^2}} \tag{3.6}$$

$$V_{out} = V_{in}\frac{R}{\sqrt{R^2 + \frac{1}{(\omega C)^2}}} \tag{3.7}$$

An important frequency for all First order circuits is the corner frequency (f_c). At the corner frequency, the resistive components of the circuits impedance are equal to the reactive components; the circuit outputs half of the power that is input. The corner frequency is also the point at which the gain of the circuit is -3dB, hence it is sometimes referred to as the -3dB point. Equations **3.8** and **3.9** can be used to calculate the corner frequency for RL and RC circuits respectively [1, pp. 49, 50].

$$f_c = \frac{R}{2\pi L} \tag{3.8}$$

$$f_c = \frac{1}{2\pi RC} \tag{3.9}$$

Theoretically, the Gain of a First order circuit should change at a rate of 20dB per frequency decade after the corner frequency when shown on a Bode Plot (Frequency in Log Scale against Gain).

3.3 First Order Circuits – Transient Response

These experiments show the transient response of a Capacitor or Inductor in series with a Resistor. To theoretically characterise the response of these circuits to a step change in voltage we can use the general equation **3.10**.

$$V = V_{initial} + \left(V_{final} - V_{initial}\right)e^{-\frac{t}{\tau}} \tag{3.10}$$

The time constant τ is a characteristic specific to each transient circuit. Equations **3.11** and **3.12** can be used to calculate the time constant for RL and RC circuits respectively [1, p. 22].

$$\tau = \frac{L}{R} \quad 3.11$$

$$\tau = RC \quad 3.12$$

Using an oscilloscope, the rise and fall time of a transient circuit can be measured. The rise and fall times are defined as the time it takes the current or voltage to get from 10% to 90% of their maximum value or vice versa. Equation **3.13** relates the rise or fall time to the circuit time constant.

$$t_{rise} \; or \; t_{fall} = 2.2\tau \quad 3.13$$

3.4 Second Order Circuits – Frequency Response

Finally, these experiments show the frequency response of a Capacitor, Inductor and Resistor in series or parallel. To theoretically characterise the response of these circuits to a frequency range we must first derive equations for their impedance using equations **3.1** and **3.2**. Equations **3.14** and **3.15** can be used to calculate the impedance for a Series RLC and Parallel RLC circuit respectively [1, pp. 52, 53].

$$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} \quad 3.14$$

$$Z = \sqrt{R^2 + \left(\frac{L}{C}\right)^2 \left(\frac{1}{\omega L - \frac{1}{\omega C}}\right)^2} \quad 3.15$$

The impedance equations can be used to calculate the voltage across components within the Second Order circuit. Consequently, they can be used to calculate the Gain for a specific Second Order Circuit using equation **3.3**. This report calculates the Gain using the Resistor Voltage as V_{out} and the Supply Voltage as V_{in}. V_{out} can be calculated for Series and Parallel RLC circuits using equations **3.16** and **3.17** respectively.

$$V_{out} = V_{in} \frac{R}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} \quad 3.16$$

$$V_{out} = V_{in} \frac{R}{\sqrt{R^2 + \left(\frac{L}{C}\right)^2 \left(\frac{1}{\omega L - \frac{1}{\omega C}}\right)^2}} \quad 3.17$$

An important frequency for Second Order circuits is the resonant frequency (f_0). The resonant frequency is the frequency at which the impedance of the Inductor is equal to that of the Capacitor. At this frequency series Second Order circuits will exhibit their maximum Gain and parallel Second Order circuits will exhibit their minimum Gain. Equation **3.18** can be used to calculate the resonant frequency for both series and parallel Second Order circuits [1, p. 53].

$$f_0 = \frac{1}{2\pi\sqrt{LC}} \quad 3.18$$

3.5 Second Order Circuits – Q Factor

At resonance the voltages across the Capacitor and Inductor in Second Order circuits directly cancel each other out. However, these voltages can be very large. Thus, they provide a magnification that can be used to amplify an input signal. The voltage across a Capacitor or Inductor in Second Order circuits can be calculated using equation **3.19**.

$$V_C = V_L = QV \tag{3.19}$$

Q is the magnification factor and is a measure of the level of amplification of the input voltage to reach the voltage across the Inductor or Capacitor when the circuit is at resonance. The magnification factor can be calculated using equation **3.20** and **3.21** for series and parallel Second Order circuits respectively [1, p. 53].

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{f_0}{\Delta f} \quad 3.20 \qquad Q = R\sqrt{\frac{C}{L}} = \frac{f_0}{\Delta f} \quad 3.21$$

The bandwidth Δf is the difference between the frequencies where the gain of the Second Order circuit is -3dB.

4 Method

The following section details the methods used to analyse the transient and frequency response of First and Second order circuits. All circuit diagrams were generated using KiCAD [2], an open source Schematic and PCB Design tool.

4.1 First Order Circuits – Frequency Analysis

Initially, a circuit was constructed according to **Figure 4.1**. The AC Source was provided by a Waveform Generator set to output a 1V Peak Sine wave, this allowed the frequency of the input Voltage to be varied.

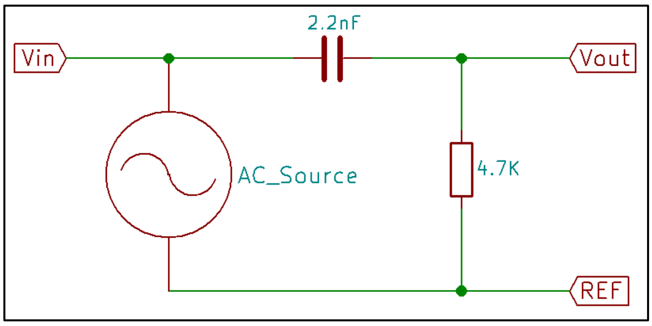

Figure 4.1: First Order RC Circuit Schematic generated using KiCAD

Initially a wide frequency range of 1kHz to 500kHz was applied to the circuit to characterise its general behaviour; multiples of 1, 2, 5 and 10 kHz were used within this frequency range. At each frequency the Voltage In (RMS Voltage between Vin and REF) and Voltage Out (RMS Voltage between Vout and REF) was measured using a Digital Multimeter (DMM) and recorded in a table.

Equation **3.3** from section **3.1** was then used to calculate the Gain at each frequency. These Gain measurements show a smaller frequency range that contains the corner frequency. Further measurements were taken at smaller frequency intervals within this range to better characterise the operation of the circuit. Bode plots of frequency vs gain were generated using these results. The same set of measurements was then repeated with the Capacitor replaced with a 4.7mH Inductor.

4.2 First Order Circuits – Transient Analysis

Transient analysis was completed using the circuit from **Figure 4.1**. The AC Source was provided by a Waveform Generator set to output a 5V Square wave at a frequency of 10kHz. The Voltage In (Voltage between Vin and REF) and Voltage Out (Voltage between Vout and REF) were measured on separate channels of an oscilloscope. Using the oscilloscope measurement functions the rise and fall times of the output Voltage were measured and recorded. The same measurements were then completed with the Capacitor replaced with a 4.7mH Inductor.

4.3 Second Order Circuits – Frequency Analysis

Initially, a series Second Order circuit was constructed according to **Figure 4.2**. The AC Source was provided by a Waveform Generator set to output a 1V RMS Sine Wave, this allowed the frequency of the input Voltage to be varied.

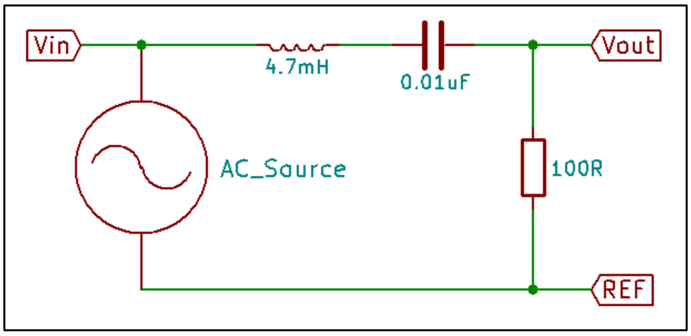

Figure 4.2: Series RLC Second Order Circuit Schematic generated with KiCAD

A wide frequency range of 1kHz to 50kHz was measured at multiples of 1, 2, 5 and 10kHz frequencies. At each frequency the Voltage In (RMS Voltage between Vin and REF) and Voltage Out (RMS Voltage between Vout and REF) were measured using a DMM and recorded. This characterised the general behaviour of the circuit and allowed a small frequency range containing the resonant frequency to be identified. Further measurements within this smaller frequency range were then taken at smaller intervals to further characterise the operation of the circuit around the resonant frequency. When the resonant frequency was reached, the RMS voltages across the Capacitor and Inductor were also recorded. Equation **3.3** from section **3.1** was used to calculate the gain of the circuit at each frequency. A Bode Plot of frequency vs gain was then generated using these results. These steps were then repeated with the 100Ω Resistor replaced with a 1200Ω Resistor.

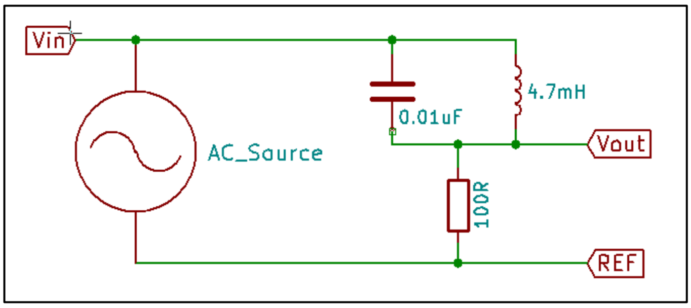

Figure 4.3: Parallel RLC Second Order Circuit Schematic generated with KiCAD

A parallel Second Order circuit was then constructed according to **Figure 4.3**. The same steps as above were then used to produce Bode Plots of frequency vs gain for the parallel Second Order circuit.

5 Results

The following section details the results obtained using the method for each circuit analysis as described in section **4.** All graphs shown were generated using Python XY (a scientific Python package) [3] with the matplotlib graphing library [4]. Tables of data can be viewed in the Appendices.

5.1 First Order Circuits – Frequency Response

Figure 5.1 shows the Gain of a First Order RC circuit (as detailed in section **4.1**) over a frequency range of 1kHz to 500kHz. The frequency is plotted on a $\log_{10}$ scale. The theoretical data was generated using equations **3.7** and **3.3**.

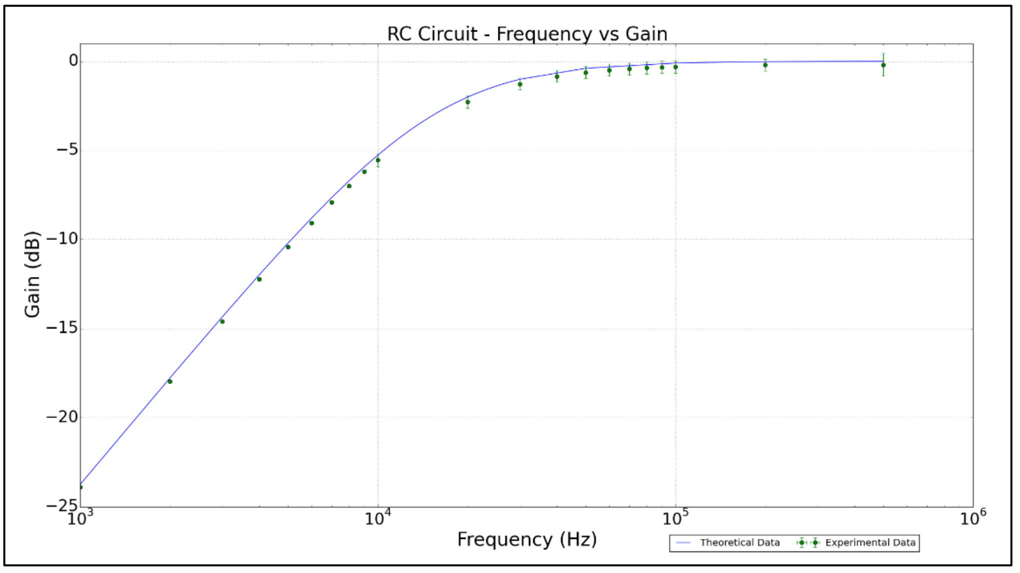

Figure 5.1: First Order RC Circuit - Frequency vs Gain Bode Plot

Figure 5.2 shows the Gain of a First Order RL circuit (as detailed in section **4.1**) over a frequency range of 1kHz to 500kHz. The frequency is plotted on a $\log_{10}$ scale. The theoretical data was generated using equations **3.6** and **3.3**.

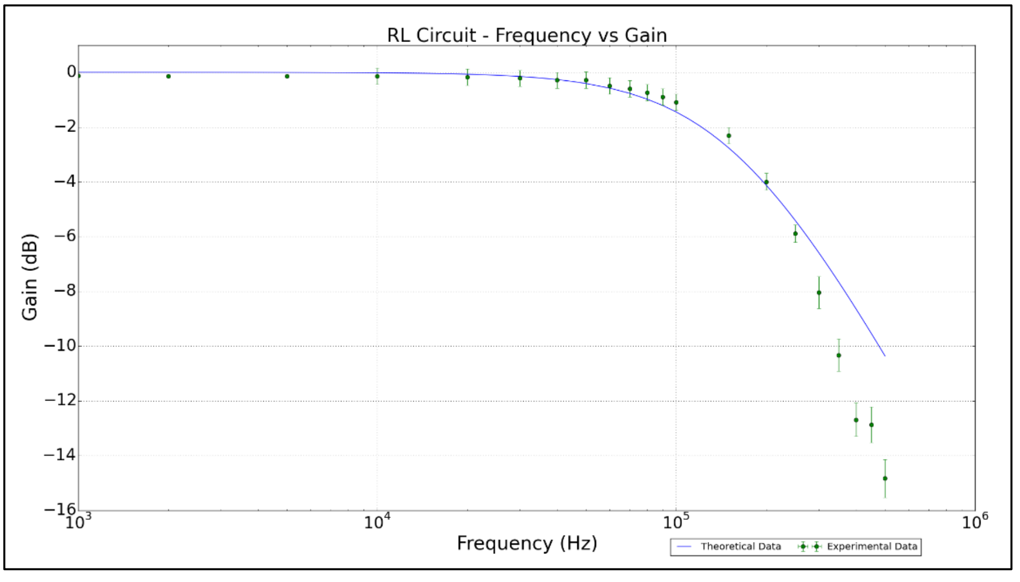

Figure 5.2: First Order RL Circuit - Frequency vs Gain Bode Plot

As detailed in section **3.2**, the corner frequency can be found by looking at the frequency that produces -3dB Gain for the circuit. The corner frequencies for the RL and RC circuits were found from **Figure 5.2** and **Figure 5.1** respectively. Equations **3.8** and **3.9** were then used to calculate the Inductance and Capacitance of each circuit respectively. The calculated values are shown in the table below.

First Order Circuit Properties – Frequency Response			
Name	**Corner Frequency**	**Calculated Component Value**	**Actual Component Value**
RC Circuit	16.7kHz	2.03nF	2.2nF
RL Circuit	169kHz	4.42mH	4.7mH

5.2 First Order Circuits – Transient Response

Using the method detailed in section **4.2** the rise and fall times for each First Order circuit were obtained using an oscilloscope. Equation **3.13** was used to calculate the circuit time constant from the rise and fall time average. Equations **3.11** and **3.12** were then used to calculate the Inductance and Capacitance of each circuit respectively. The results from these calculations can be seen in the table below.

First Order Circuit Properties – Transient Response				
Name	**Rise Time**	**Fall Time**	**Calculated Component Value**	**Actual Component Value**
RC Circuit	21µS ±0.5µS	20µS ±0.5µS	1.98nF	2.2nF
RL Circuit	1.85µS ±0.05µS	1.95µS ±0.05µS	4.06mH	4.7mH

5.3 Second Order Circuits – Frequency Response

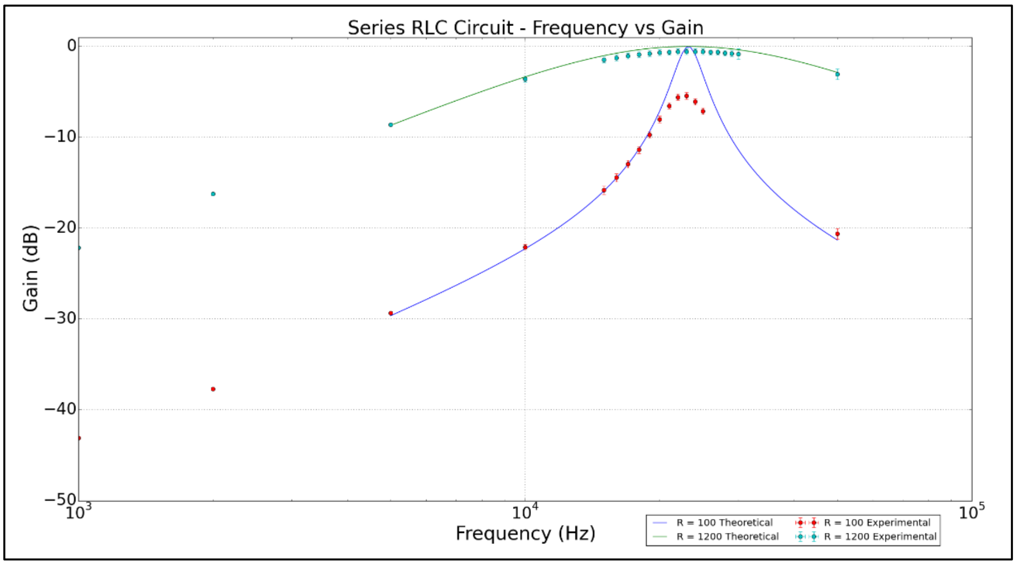

Figure 5.3: Series RLC Second Order Circuit – Frequency vs Gain Bode Plot

Figure 5.3 shows the Gain of a series Second Order RLC circuit (as detailed in section **4.3**) over a frequency range of 1kHz to 50kHz. The frequency is plotted on a $\log_{10}$ scale. The theoretical data was generated using equations **3.16** and **3.3** respectively.

Figure 5.4 shows the Gain of a parallel Second Order RLC circuit (as detailed in section **4.3**) over a frequency range of 1kHz to 50kHz. The frequency is plotted on a $\log_{10}$ scale. The theoretical data was generated using equations **3.17** and **3.3** respectively.

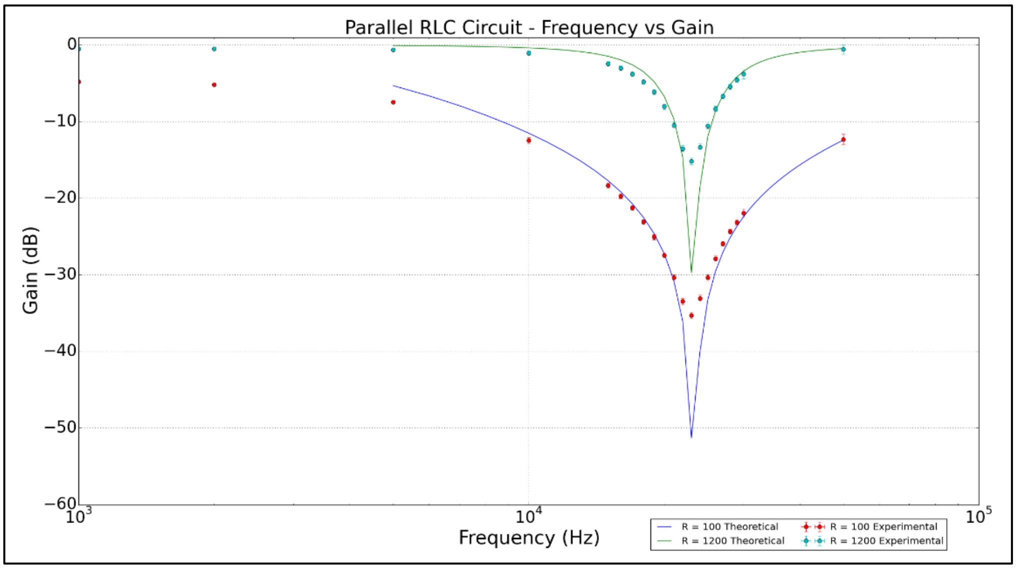

Figure 5.4: Parallel RLC Second Order Circuit - Frequency vs Gain Bode Plot

The resonant frequency can be found by looking at the maximum or minimum Gain on the series or parallel Second Order Circuit Bode Plot respectively. The resonant frequency can then be used with equation **3.20** or **3.21** to calculate the magnification of the Second Order circuit. In this case, equation **3.19** was used with the voltages across the Capacitor and Inductor to calculate the Q Factor for the Series RLC Circuit. Equations **3.20** and **3.21** can also be used to calculate the theoretical magnification factor of the circuit using component values. The table below shows a comparison between the theoretical magnification factor and the experimental magnification factor calculated from the empirical results for the Series RLC Circuit as well as the resonant frequency for the Parallel RLC Circuit.

Second Order Circuits – Magnification Factors			
Name	**Resonant Frequency**	**Experimental Q Factor**	**Theoretical Q Factor**
Series 100Ω Resistor	22.5kHz	3.630	6.856
Series 1200Ω Resistor	23kHz	0.530	0.571
Parallel 100Ω Resistor	22.7kHz	-	0.146
Parallel 1200Ω Resistor	22.7kHz	-	1.750

6 Discussion

This section discusses the differences between the theoretical models and the empirical data generated from the experiments detailed in section **4**. Sources of error within the circuits will be discussed and the potential causes of the difference between the theoretical and empirical data will be explored.

6.1 Sources of Uncertainty within Experiments

The results produced by the experiments tend exhibit the same general behaviour as would be expected from the theoretical model. This is largely due to the high accuracy of the test equipment being used allowing for very accurate Input Signals (Waveform Generator) to be applied to the circuit and the accurate measurement of Output Signals (DMM).

However, there is a level of uncertainty that must be considered when analysing the empirical data, as is shown by the error bars on the Bode Plots. The main source of this uncertainty was the DMM, which has an uncertainty that increases with input signal frequency. For First Order circuits, any input signal with a frequency of 30kHz or greater produces an uncertainty of 3% of Reading plus 0.3% of Range [5], this explains the large error bars on these graphs at high frequency. For Second Order circuits, any input signal with a frequency in the 10kHz to 30kHz range has an error of 1.5% of Reading plus 0.3% of Range [5]. This is the main contributor to the uncertainty of the measurement of the resonant frequency.

6.2 First Order Circuits – Frequency Response Discussion

It is clear from the trend of both the theoretical and empirical data that the RC Circuit is acting as a High Pass Filter. At low input signal frequencies, the circuit shows a high level of attenuation (approximately -24dB at 1kHz); at high input signal frequencies the circuit shows a low level of attenuation (approximately -0.5dB at frequencies greater than 100kHz). This conclusion is backed up by equation **3.2** which shows that the impedance of the capacitor will decrease as the input frequency increases [1, p. 45]. Generally, the gain decreases at a rate of 20dB per decade, as would be expected from the theoretical model.

Generally, both the trend shown by the empirical data and the calculated corner frequency agree with the behaviour described by equations **3.7** and **3.9**. This is backed up by the 2.02nF Capacitance value calculated from the corner frequency which is well within the tolerance range provided by the errors in measurement and errors in the actual Capacitance value of the component. This indicates that the testing methodology was successful and that system uncertainties were at a low enough level as to not affect the output of the circuit.

The First Order RC Circuit shows a small but significant offset of approximately -0.5dB from the 0dB point even when the input frequency is well above the corner frequency. As this offset appears uniformly across the entire dataset it is most likely caused by a parasitic resistance within the circuit that forms a potential divider with the resistor even when the reactive impedance is very low. It is likely that this parasitic resistance is introduced by the capacitor, however the waveform generator is also introducing its internal impedance to the circuit. This parasitic resistance will have a small voltage drop across it; consequently, it is not possible for the entire source voltage to be dropped across the Resistor.

According to the trend of the theoretical and empirical data, the First Order RL Circuit is acting as a Low Pass Filter. At low signal frequencies, the circuit shows a low level of attenuation (approximately -0.2dB at frequencies less than 20kHz); at high input signal frequencies the circuit shows a high level of attenuation (approximately -10.5dB at 350kHz). This conclusion is backed up by equation **3.1** which shows that the impedance of the inductor will increase with an increase in input frequency.

At frequencies below 100kHz the empirical and theoretical data follow the same trend with the error bars on the empirical data overlapping the line produced by the theoretical model. However, at frequencies of 100kHz and above the data moves away from the theoretical line, showing a greater attenuation than the theoretical model would predict. One explanation for this behaviour would be a parasitic series resistance induced into the circuit. It is likely that this resistance is introduced by the Inductor, however the Waveform Generator will also introduce its internal impedance. On the contrary, the Inductance value of 4.42mH calculated from the corner frequency is well within the tolerance values of the measurement system and the Inductance value of the component used within the circuit.

6.3 First Order Circuits – Transient Response Discussion

Analysis of Rise and Fall times with the Oscilloscope gave an empirical value of 1.98nF for the Capacitance of the RC circuit. This is further away from the actual Capacitance value of 2.2nF than the value generated using the corner frequency, however it is still within the tolerance bands of the measurement system for the circuit and the component tolerance. As the oscilloscope was used for these measurements a larger uncertainty can be applied to the measurement of the rise and fall times as the oscilloscope measures voltage with a greater uncertainty than the DMM. Use of the Oscilloscope measurement functions will also add a level of uncertainty.

An empirical value of 4.06mH was calculated for the Inductance of the RL circuit. Again, this is further away from the actual Inductance value of 4.7mH than the value generated using the corner frequency. This error is introduced by the greater uncertainty in the oscilloscope voltage measurement and the use of the oscilloscope measurement functions to measure the rise and fall time.

6.4 Second Order Circuits – Frequency Response Discussion

Both the empirical results for the series Second Order Circuit and the parallel Second Order Circuit follow the curves generated by the theoretical models very closely, with resonant frequencies within the same range and the same general behaviour across the entire frequency range.

For the series RLC Circuit, the negative offset of gain around low, high and resonant frequencies is caused by a parasitic resistance in the circuit introduced by the Capacitor and Inductor. This creates a potential divider circuit that prevents the circuit from reaching the theoretical zero gain at these frequencies.

For the parallel RLC Circuit, the positive offset of gain around the resonant frequency is caused by the parasitic resistance of the Capacitor and Inductor lowering the overall impedance of the parallel combination. This causes a greater voltage drop across the resistor than the theoretical model predicts, as the reactive section of the circuit has a lower overall impedance than the model predicts at a given frequency.

The error in the magnification factors from the empirical data for the series RLC Circuit is due to the difficulty in finding the exact resonant frequency of the circuit. This means the voltages measured across the Capacitor and Inductor will have a significant error as they were not measured at the circuits exact resonant frequency. Consequently, this causes an error in the magnification factor calculated from the empirical data. This is backed up by the calculated magnification factors being lower than the theoretical magnification factors. The voltage across the Capacitor and Inductor will be at their maximum at resonance. Consequently, any voltage measured across these components at a frequency slightly above or below resonance will be lower than the maximum and thus produce a lower calculated magnification factor.

Ultimately, we can state that the series RLC Circuits act as Band Pass Filters. They allow a small band of frequencies to pass with very little attenuation, whilst the frequencies outside this range are heavily attenuated and thus damped.

The parallel RLC Circuits act as Notch Filters. They produce a massive attenuation on frequencies within a certain range, causing them to be damped from the circuit. Any frequencies outside this range can pass through this range with little attenuation.

7 Conclusion

To conclude the information gained from the experiments on First Order circuits, we can say the following. When used in series with a Resistor, Capacitors and Inductors can be used to produce frequency filters that low frequencies above or below a certain critical frequency to pass with little attenuation. These filters are known as High Pass and Low Pass Filters. These circuits can also be used as timers by applying a step change in input voltage. The time constant of these circuits can be changed by changing component values within the circuit according to the specific time constant of the first order circuit.

Second Order circuits have a specific resonant frequency which can be tuned by changing the values of the reactive components within the circuit. These circuit can be used to produce filters that allow or reject input frequencies within a certain band, known as Band Pass and Notch Filters respectively.

The use of Bode Plots is key in the characterisation of filter circuits as they clearly show the frequencies at which a circuit will produce large attenuation of the input signal. They aid in the tuning of resonant circuits to specific frequency bands and show the specific characteristics of each circuit such as the corner frequency, resonant frequency and magnification factor.

Finally, when testing any circuit, the effects of component tolerance and parasitic values introduced by measurement equipment or components must always be considered. As is shown by the Bode Plots in sections **5.1** and **5.3** they can produce offsets in the data produced by the experiment that must be accounted for in the discussion of the results. If these offsets are too large to make the data useful then the experiment must be rerun with changes that ensure that these offsets are minimised.

8 References

[1] P. Horowitz and W. Hill, The Art of Electronics, 3 ed., New York, NY: Cambridge University Press, 2015.

[2] J.-P. Charras, D. Hollenbeck and W. Stambaugh, "KiCAD EDA," KiCAD, August 2017. [Online]. Available: http://kicad-pcb.org/.

[3] P. Raybaut and G. Davar, "Python XY," [Online]. Available: https://python-xy.github.io/.

[4] J. D. Hunter, "Matplotlib," [Online]. Available: https://matplotlib.org/.

[5] Agilent Technologies, Agilent 34450A 5 1/2 Digit Multimeter User's Guide, Santa Clara, CA: Agilent Technologies, 2014.

9 Appendices

9.1 RC Circuit - Frequency Response Data Table

RC Circuit - Gain vs Frequency Response					
Frequency (Hz)	Voltage In (V)	Voltage Out (V)	Gain (dB)	Minimum Error (dB)	Maximum Error (dB)
1000	0.705	0.045	-23.89953206	-23.96592502	-23.83319246
2000	0.705	0.089	-17.97598221	-18.03279743	-17.91915023
3000	0.704	0.131	-14.60602727	-14.65972172	-14.55229753
4000	0.702	0.172	-12.2161733	-12.31400503	-12.11877008
5000	0.704	0.212	-10.42473596	-10.51293474	-10.33682746
6000	0.7	0.246	-9.083258658	-9.165828593	-9.000906134
7000	0.696	0.28	-7.909024165	-7.987352617	-7.830862741
8000	0.694	0.31	-6.999955532	-7.075301495	-6.924743936
9000	0.692	0.339	-6.198127925	-6.271100773	-6.125264803
10000	0.703	0.371	-5.551628308	-5.920266789	-5.184395331
20000	0.7	0.538	-2.286315287	-2.632821045	-1.940256523
30000	0.699	0.604	-1.268804742	-1.609960748	-0.927879013
40000	0.698	0.634	-0.835323295	-1.174448943	-0.496344787
50000	0.698	0.65	-0.61884132	-0.956934962	-0.280854995
60000	0.698	0.66	-0.486229742	-0.823703849	-0.148839157
70000	0.699	0.666	-0.420058932	-0.757117871	-0.083071681
80000	0.667	0.64	-0.358917199	-0.699351833	-0.018547171
90000	0.673	0.648	-0.328801167	-0.668381427	0.010720683
100000	0.7	0.675	-0.315885344	-0.65236	0.020535895
200000	0.7	0.683	-0.213546727	-0.549560259	0.122430956
500000	0.513	0.502	-0.188272959	-0.812437144	0.43581118

9.2 RL Circuit - Frequency Response Data Table

RL Circuit - Gain vs Frequency Response					
Frequency (Hz)	Voltage In (V)	Voltage Out (V)	Gain (dB)	Minimum Error (dB)	Maximum Error (dB)
1000	1.392	1.372	-0.125702478	-0.173017088	-0.078388364
2000	1.392	1.371	-0.13203561	-0.17935485	-0.08471689
5000	1.392	1.371	-0.13203561	-0.17935485	-0.08471689
10000	1.363	1.341	-0.14134156	-0.440502619	0.157808695
20000	1.361	1.334	-0.174045912	-0.473338344	0.125233164
30000	1.386	1.352	-0.215730773	-0.514419072	0.082941264
40000	1.386	1.341	-0.286689048	-0.585538215	0.012138408
50000	1.371	1.328	-0.276787595	-0.576032541	0.022436145
60000	1.386	1.311	-0.483210772	-0.782512406	-0.183946204
70000	1.388	1.295	-0.602393954	-0.901918854	-0.302915552
80000	1.39	1.276	-0.743282517	-1.043085764	-0.443537086
90000	1.392	1.255	-0.899910189	-1.200034709	-0.599856284
100000	1.4	1.234	-1.09625752	-1.396636477	-0.795965155
150000	1.427	1.094	-2.308133022	-2.610916402	-2.005543376
200000	1.474	0.931	-3.990956051	-4.297413505	-3.684862241
250000	1.513	0.768	-5.88955416	-6.201616269	-5.578091834
300000	1.545	0.612	-8.043541232	-8.625216214	-7.463586868
350000	1.549	0.472	-10.32218838	-10.91691678	-9.730084449
400000	1.534	0.356	-12.68750723	-13.30108257	-12.07789298
450000	1.127	0.256	-12.87367901	-13.52298257	-12.23023495
500000	0.998	0.181	-14.82903933	-15.52536183	-14.14211112

9.3 Series RLC Circuit 100Ω Resistor – Frequency Response Data Table

Series RLC Circuit - Resistor Gain vs Frequency Response (100 Ohm Resistor)					
Frequency (Hz)	Voltage In (V)	Resistor Voltage (V)	Gain (dB)	Minimum Error (dB)	Maximum Error (dB)
1000	0.998	0.007	-43.08065003	-43.24930647	-42.91421932
2000	0.998	0.013	-37.70374378	-37.81437674	-37.59384853
5000	0.996	0.034	-29.33560843	-29.40468676	-29.26666378
10000	0.991	0.078	-22.07958104	-22.40002684	-21.7593976
15000	0.97	0.157	-15.81744164	-16.27461996	-15.36753036
16000	0.96	0.182	-14.4439969	-14.8778786	-14.01587511
17000	0.946	0.213	-12.95023066	-13.36301861	-12.54192478
18000	0.926	0.25	-11.37341956	-11.76816362	-10.98211965
19000	0.897	0.293	-9.718496454	-10.09842702	-9.341177114
20000	0.859	0.342	-7.999341156	-8.367471469	-7.633149844
21000	0.817	0.386	-6.512695037	-6.873464309	-6.153402334
22000	0.786	0.414	-5.568444098	-5.925777322	-5.212334624
23000	0.782	0.418	-5.440609426	-5.797493102	-5.084916342
24000	0.804	0.399	-6.085663061	-6.444688134	-5.727995593
25000	0.836	0.368	-7.127169175	-7.490606404	-6.765387253
50000	0.9	0.084	-20.59926447	-21.18067803	-20.01798942

9.4 Series RLC Circuit 1200Ω Resistor – Frequency Response Data Table

Series RLC Circuit - Gain vs Frequency Response (1200 Ohm Resistor)					
Frequency (Hz)	Voltage In (V)	Resistor Voltage (V)	Gain (dB)	Minimum Error (dB)	Maximum Error (dB)
1000	0.998	0.078	-22.14071877	-22.19530921	-22.08614362
2000	0.997	0.154	-16.22348875	-16.32362221	-16.12390358
5000	0.991	0.369	-8.580945767	-8.648050498	-8.51395508
10000	0.978	0.649	-3.56188316	-3.889547757	-3.23472781
15000	0.966	0.815	-1.476390354	-1.796042479	-1.156922142
16000	0.964	0.836	-1.237415129	-1.556303833	-0.918678368
17000	0.963	0.854	-1.043368329	-1.361615218	-0.725247983
18000	0.962	0.868	-0.893106937	-1.210880048	-0.57544115
19000	0.961	0.879	-0.774690252	-1.092108311	-0.457364639
20000	0.96	0.888	-0.677165345	-0.994305063	-0.360105992
21000	0.96	0.894	-0.618674285	-0.935613376	-0.301808321
22000	0.959	0.898	-0.57084541	-0.887680043	-0.254078112
23000	0.959	0.899	-0.561178309	-0.877980061	-0.244442707
24000	0.959	0.899	-0.561178309	-0.877980061	-0.244442707
25000	0.959	0.898	-0.57084541	-0.887680043	-0.254078112
26000	0.959	0.895	-0.599911437	-0.916845153	-0.283048626
27000	0.959	0.891	-0.638818063	-0.955884929	-0.321826902
28000	0.96	0.886	-0.696750223	-1.013957421	-0.379625826
29000	0.961	0.88	-0.76481431	-1.08219805	-0.447521776
30000	0.961	0.873	-0.834182879	-1.412604474	-0.255943407
50000	0.976	0.688	-3.037227589	-3.623553526	-2.45165616

9.5 Parallel RLC Circuit 100Ω Resistor – Frequency Response Data Table

Parallel RLC Circuit - Gain vs Frequency Response (100 Ohm Resistor)					
Frequency (Hz)	Voltage In (V)	Resistor Voltage (V)	Gain (dB)	Minimum Error (dB)	Maximum Error (dB)
1000	0.77	0.444	-4.782155101	-4.847773448	-4.716599296
2000	0.784	0.431	-5.19677585	-5.262786023	-5.130834602
5000	0.85	0.36	-7.462328499	-7.531473949	-7.393293685
10000	0.935	0.223	-12.45013496	-12.85759274	-12.04683155
15000	0.972	0.118	-18.31568515	-18.62510147	-18.00610042
16000	0.976	0.101	-19.70256888	-20.01566137	-19.38944398
17000	0.979	0.085	-21.22727532	-21.54523414	-20.90946437
18000	0.982	0.069	-23.06524794	-23.39037277	-22.74054327
19000	0.984	0.055	-25.05264818	-25.38752807	-24.71857317
20000	0.987	0.042	-27.42135725	-27.77114169	-27.07300553
21000	0.988	0.03	-30.3527138	-30.72788793	-29.98014395
22000	0.989	0.021	-33.45953994	-33.87295177	-33.05075565
23000	0.989	0.017	-35.2949474	-35.73848853	-34.85784377
24000	0.989	0.022	-33.05547222	-33.46307564	-32.65216937
25000	0.989	0.03	-30.36150074	-30.73664868	-29.98895805
26000	0.989	0.04	-27.86272601	-28.21563094	-27.51139429
27000	0.989	0.05	-25.92452575	-26.26411216	-25.58594552
28000	0.989	0.06	-24.34090082	-24.67161956	-24.010832
29000	0.988	0.069	-23.11815708	-23.44312362	-22.79361651
30000	0.987	0.079	-21.93380123	-22.51477648	-21.35326649
50000	0.958	0.232	-12.31755048	-12.98179808	-11.65978668

9.6 Parallel RLC Circuit 1200Ω Resistor – Frequency Response Data Table

Parallel RLC Circuit - Gain vs Frequency Response (1200 Ohm Resistor)					
Frequency (Hz)	Voltage In (V)	Resistor Voltage (V)	Gain (dB)	Minimum Error (dB)	Maximum Error (dB)
1000	0.959	0.906	-0.49380819	-0.547197856	-0.440421781
2000	0.959	0.905	-0.503400559	-0.556800852	-0.450003589
5000	0.96	0.896	-0.599264468	-0.65275206	-0.545780855
10000	0.963	0.854	-1.043368329	-1.361615218	-0.725247983
15000	0.97	0.731	-2.457087146	-2.780374766	-2.134126339
16000	0.972	0.686	-3.026842984	-3.352460515	-2.701644274
17000	0.975	0.631	-3.779505129	-4.108418594	-3.451143065
18000	0.978	0.564	-4.781195016	-5.11503397	-4.448107095
19000	0.981	0.484	-6.136472915	-6.47803034	-5.795986656
20000	0.985	0.391	-8.025189462	-8.379733625	-7.672282421
21000	0.987	0.295	-10.48990273	-10.86661318	-10.11586951
22000	0.989	0.209	-13.50100011	-13.91502358	-13.09163903
23000	0.99	0.173	-15.15178183	-15.5925495	-14.71727969
24000	0.99	0.214	-13.30442842	-13.71542514	-12.89792487
25000	0.988	0.294	-10.52819228	-10.90518469	-10.15389321
26000	0.986	0.379	-8.304754099	-8.661431217	-7.949811318
27000	0.984	0.457	-6.661577967	-7.006306153	-6.318059956
28000	0.982	0.525	-5.439043688	-5.776280043	-5.102703177
29000	0.979	0.582	-4.517194143	-4.849547936	-4.185534508
30000	0.977	0.631	-3.797304089	-4.38714735	-3.208453894
50000	0.961	0.899	-0.579273919	-1.15680245	-0.001869803